Name: ______________________________

Math 10560, Practice Final Exam: May 5, 2024

Instructor: __________________________

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- The exam lasts for two hours.
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1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(6 pts.) Let $f(x) = e^x - 1$ and let f^{-1} denote the inverse function. Then $(f^{-1})'(e^2 - 1) =$ is

(a) e^{-1} (b) $\dfrac{1}{e^2 - 1}$ (c) e

(d) e^2 (e) e^{-2}

2.(6 pts.) Solve the following equation for x:

$$\ln(x + 4) - \ln x = 1 \ .$$

(a) $x = \dfrac{4}{1 - e}$ (b) $x = \dfrac{4}{e - 1}$ and $x = \dfrac{4}{e + 1}$

(c) There is no solution. (d) $x = e + 2$ and $x = e - 2$

(e) $x = \dfrac{4}{e - 1}$

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3.(6 pts.) Find the derivative of $(x^2+1)^{x^2+1}$.

(a) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

(b) $(x^2+1)^{x^2+1}\ 2x\big(\ln(x^2+1)+1\big)$

(c) $2x(x^2+1)^{x^2}$

(d) $(x^2+1)^{x^2+1}$

(e) This function is not defined and hence has no derivative.

4.(6 pts.) $\lim\limits_{x\to 0^+}(\cos x)^{\frac{1}{x^2}} =$

(a) $e^{-\frac{1}{2}}$ (b) 1 (c) Does not exist

(d) ∞ (e) e

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5.(6 pts.) The integral

$$\int_0^{\pi/2} x\cos(x)dx$$

is

(a) divergent (b) $\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$ (c) $\frac{\pi}{2} - 1$

(d) 0 (e) $1 - \frac{\pi}{2}$

6.(6 pts.) Evaluate

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx.$$

(a) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x}{3}\right] + C$ (b) $\frac{1}{2}x\sqrt{9-x^2} + C$

(c) $9\arcsin(x/3) + C$ (d) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x\sqrt{9-x^2}}{9}\right] + C$

(e) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x^2}{9}\right] + C$

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7.(6 pts.) If you expand $\dfrac{2x+1}{x^3+x}$ as a partial fraction, which expression below would you get?

(a) $\dfrac{1}{x} + \dfrac{-x+2}{x^2+1}$

(b) $\dfrac{-1}{x^2} + \dfrac{1}{x+1}$

(c) $\dfrac{2}{x} + \dfrac{1}{x^2+1}$

(d) $\dfrac{-1}{x} + \dfrac{x}{x^2+1}$

(e) $\dfrac{-2}{x} + \dfrac{1}{x^2+1}$

8.(6 pts.) The integral

$$\int_0^2 \frac{1}{1-x}dx$$

is

(a) $\dfrac{\pi}{\sqrt{2}}$

(b) 0

(c) divergent

(d) $\dfrac{\pi}{6}$

(e) $\ln 2$

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9.(6 pts.) If 100 grams of radioactive material with a half–life of two days are present at day zero, how many grams are left at day three?

(a) $\dfrac{100}{4^{1/3}}$ (b) $\dfrac{100}{\sqrt{8}}$ (c) $\dfrac{100}{2^{1/3}}$ (d) 50 (e) $\dfrac{100}{\sqrt{2}}$

10.(6 pts.) If $x\dfrac{dy}{dx} + 3y = \dfrac{4}{x}$, and $y(1) = 10$, find $y(2)$.

(a) 0 (b) $\dfrac{1}{2}$ (c) $\dfrac{4}{3}$ (d) 7 (e) 2

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11.(6 pts.) The solution to the initial value problem

$$y' = x\cos^2 y \qquad\qquad y(2) = 0$$

satisfies the implicit equation

(a) $\tan(y) = \frac{x^2}{2} - 2$

(b) $\cos y = x - 1$

(c) $\cos(y) = x + \cos(2)$

(d) $\frac{ey}{2} = e^{\cos x} - e^{\cos 2}$

(e) $e^{2y+1} = \arcsin(x-2) + e$

12.(6 pts.) Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \qquad y(1) = 0.$$

(a) $y(1.2) \approx .112$ (b) $y(1.2) \approx .201$ (c) $y(1.2) \approx .211$

(d) $y(1.2) \approx .101$ (e) $y(1.2) \approx .111$

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13.(6 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$

(a) $\dfrac{20}{3}$ (b) $\dfrac{5}{12}$ (c) $\dfrac{5}{4}$ (d) $\dfrac{5}{3}$ (e) $\dfrac{4}{15}$

14.(6 pts.) Which of the following series converge conditionally?

$$\text{(I)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} \qquad \text{(II)} \sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\ln n} \qquad \text{(III)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n} \ ?$$

(a) (II) converges conditionally, (I) and (III) do not converge conditionally

(b) (I) and (III) converge conditionally, (II) does not converge conditionally

(c) (III) converges conditionally, (I) and (II) do not converge conditionally

(d) (II) and (III) converge conditionally, (I) does not converge conditionally

(e) (I) and (II) converge conditionally, (III) does not converge conditionally

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15.(6 pts.) Which series below absolutely converges?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$

16.(6 pts.) The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$$

is

(a) $[-4,-2)$ (b) $(-4,-2)$ (c) $(2,4)$

(d) $(-1,1)$ (e) $[2,4]$

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17.(6 pts.) If $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(2n+1)!}$, find the power series centered at 2 for the function $\int_2^x f(t)\,dt$.

(a) The given function can not be represented by a power series centered at 2.

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)(2n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n^2)(2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{2n+1}}{(n+1)(2n)!}$

18.(6 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\frac{x^2}{1+x}$?

(a) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(c) $\sum_{n=0}^{\infty}(-1)^n\, x^{n+2}$

(d) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$

(e) $\sum_{n=0}^{\infty} x^{2n+2}$

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19.(6 pts.) The following is the fourth order Taylor polynomial of the function $f(x)$ at a.

$$T_4(x) = 10 + 5(x-a) + \sqrt{3}(x-a)^2 + \frac{1}{2\pi}(x-a)^3 + 17e(x-a)^4$$

What is $f'''(a)$?

(a) $\frac{1}{6\pi}$ (b) $17e$ (c) $\frac{1}{2\pi}$ (d) $\frac{3}{\pi}$ (e) $2\sqrt{3}$

20.(6 pts.) $\lim\limits_{x\to 0} \dfrac{\sin(x^3) - x^3}{x^9} =$

Hint: Without MacLaurin series this may be a long problem.

(a) ∞ (b) 0 (c) $\dfrac{7}{9}$ (d) $-\dfrac{1}{6}$ (e) $\dfrac{9}{7}$

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21.(6 pts.) Which line below is the tangent line to the parameterized curve

$$x = \cos t + 2\cos(2t), \qquad y = \sin t + 2\sin(2t)$$

when $\quad t = \pi/2$?

(a) $\quad y = x + 3$

(b) $\quad y = -4x - 7$

(c) $\quad y = -x + 3$

(d) $\quad y = 1$

(e) $\quad y = 4x + 9$

22.(6 pts.) Which integral below gives the arclength of the curve $x = 1 - 2\cos t$, $y = \sin^2\big(t/2\big)$, $0 \le t \le \pi$?

(a) $\quad \displaystyle\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\; dt$

(b) $\quad \displaystyle\int_0^{\pi} \sqrt{4\sin^2 t + \sin^2(t/2)\cos^2(t/2)}\; dt$

(c) $\quad \displaystyle\int_0^{\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\; dt$

(d) $\quad \displaystyle\int_0^{\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\; dt$

(e) $\quad \displaystyle\int_0^{\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^4(t/2)}\; dt$

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23.(6 pts.) The point $(2, \dfrac{11\pi}{3})$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(1, -\sqrt{3}\,)$

(b) $(\sqrt{3}, -1)$

(c) $(-\sqrt{3}, 1)$

(d) $(-1, \sqrt{3}\,)$

(e) Since $\dfrac{11\pi}{3} > 2\pi$, there is no such point.

24.(6 pts.) Find the equation for the tangent line to the curve with polar equation: $r = 2 - 2\cos\theta$ at the point $\theta = \pi/2$.

(a) $y = 2 - \pi + 2x$ (b) $y = 2 + \dfrac{\pi}{2} - x$ (c) $y = 2 - x$

(d) $y = 0$ (e) $y = 2 + 2x$

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25.(6 pts.) Find the length of the polar curve between $\theta = 0$ and $\theta = 2\pi$

$$r = e^{-\theta}.$$

(a) $2e^{-4\pi}$ (b) $\frac{1}{4}(1 - e^{-4\pi})$ (c) $2 - e^{-2\pi}$

(d) $2\pi(1 + e^{-2\pi})$ (e) $\sqrt{2}(1 - e^{-2\pi})$

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

The hyperbolic sine and cosine functions are defined to be:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

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Math 10560, Practice Final Exam: Instructor: ANSWERS
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15.	(a)	(b)	(c)	(•)	(e)
16.	(•)	(b)	(c)	(d)	(e)
17.	(a)	(•)	(c)	(d)	(e)
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20.	(a)	(b)	(c)	(•)	(e)
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22.	(a)	(•)	(c)	(d)	(e)
23.	(•)	(b)	(c)	(d)	(e)
24.	(a)	(b)	(•)	(d)	(e)
25.	(a)	(b)	(c)	(d)	(•)